# Supervised Joint Nonlinear Transform Learning with Discriminative-Ambiguous Prior for Generic Privacy-Preserved Features

**Dimche Kostadinov**, Behrooz Razeghi,
Shideh Rezaeifar and Slava Voloshynovskiy

Stochastic Information Processing Group
CVML, University of Geneva
Switzerland

March 21th, 2019

UNIVERSITÉ DE GENÈVE

stochastic information processing

# Outline

Introduction

Related Work

Proposed Model

# Introduction

**Privacy-preserving content search/identification/recognition**

- Biometrics
- Physical object recognition and security
- Medical/clinical applications
- Privacy-sensitive multimedia records

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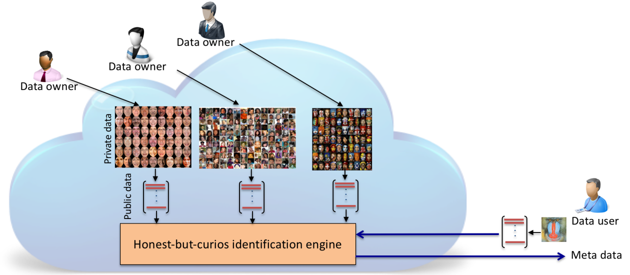

# Introduction

## Problem Formulation

**Objectives of parties**

- **Data owners** wish to share their data with "authorized" data users
- **Data users** seek some utility based on the query
- **Server** (service provider) is honest-but-curious

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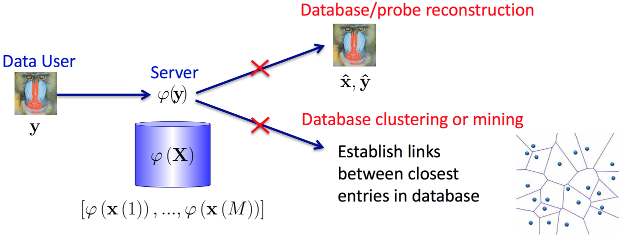
**Goal of privacy protection** in outsourced services



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# Privacy-Protected Template Generation using Sparsification with Ambiguization

**Contribution:** Model a generalized randomization technique with an information loss prior for privacy protection mechanisms

**Three Fundamental Elements**

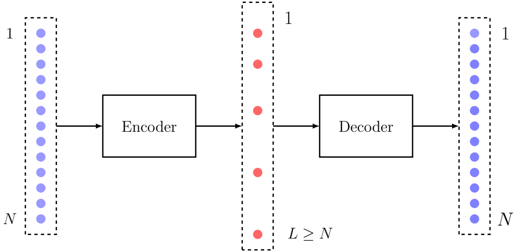
- Sparsification
- Ambiguization
- Privacy-Preserving Search

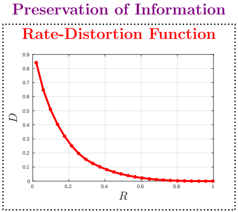
**Advantages:**

- Fast search / memory efficient
- Tunable reconstruction/recognition accuracy at server side
- Limit inference of the public data even for the authorized data users
- Server cannot reveal a structure of the database

# Fundamental Elements: Sparsification

## Main Idea



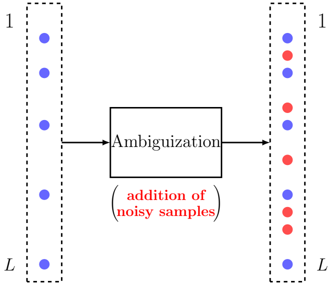

- $\mathbf{x}(m) \in \mathbb{R}^N$
- $\mathbf{x}(m) \sim p(\mathbf{x})$

- $\mathbf{u}(m) \in \{-1, 0, +1\}^L$
- $\|\mathbf{u}(m)\|_0 \leq S_x$
- Rate: $R = \frac{1}{L} \log_2 \left( \binom{L}{S_x} 2^{S_x} \right)$

- $\hat{\mathbf{x}}(m) \in \mathbb{R}^N$
- Distortion : $\frac{1}{N} \|\mathbf{x}(m) - \hat{\mathbf{x}}(m)\|_2^2 \leq D$

# Fundamental Elements: Ambiguization

## Main Idea

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- **Public Domain**
- $\mathbf{u}(m) \bigoplus \mathbf{n}$

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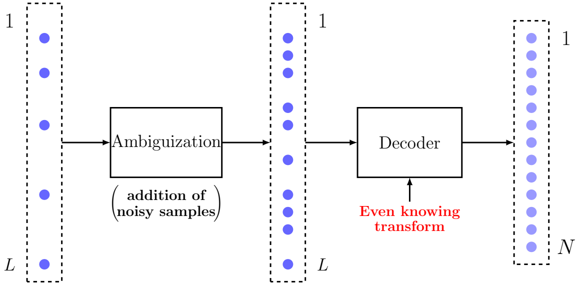


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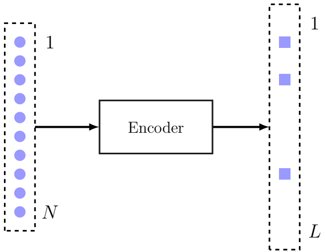
- $\hat{\mathbf{x}}(m) \in \mathbb{R}^N$
- $\|\mathbf{x}(m) - \hat{\mathbf{x}}(m)\|_2^2 \rightarrow \simeq 2N\sigma_{\mathbf{x}}^2$

- Prevent reconstruction from $\mathbf{u}(m) \bigoplus \mathbf{n}$ and from probe $\mathbf{y}$
- Preclude server from discovering the structure of the database $\mathcal{A}$

# Fundamental Elements: Privacy Preserving Search

## Main Idea: User sends only positions of interest




- $\mathbf{y} \in \mathbb{R}^N$
- $\mathbf{b} \in \{-1, 0, +1\}^L$
- $\|\mathbf{b}\|_0 \le S_y$

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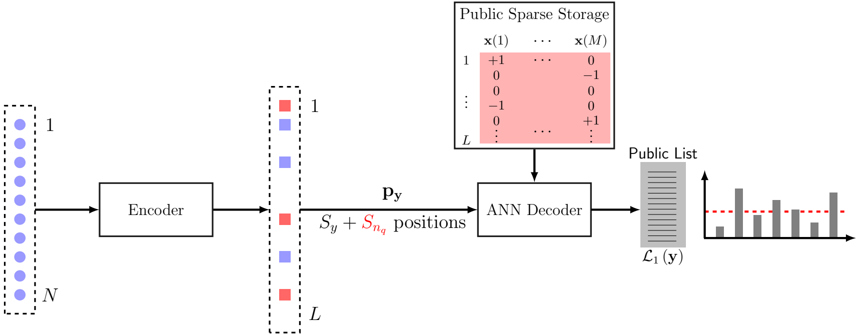


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- Add $S_{n_q}$ random positions

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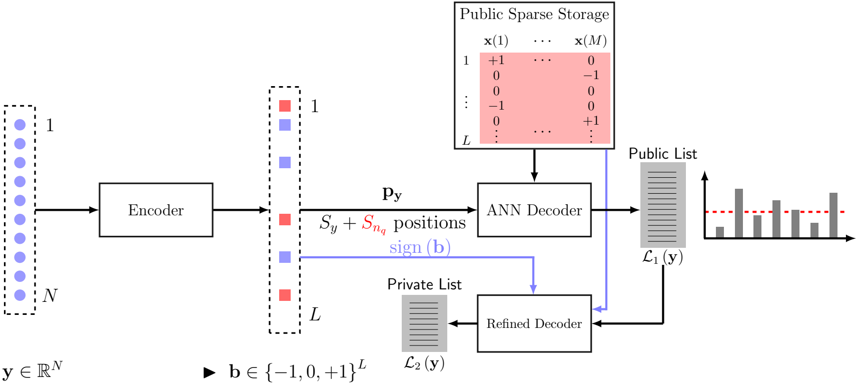


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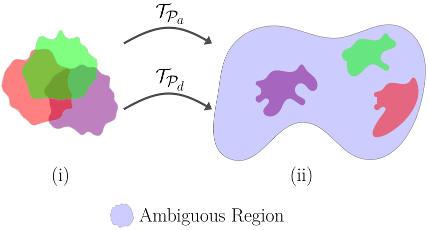
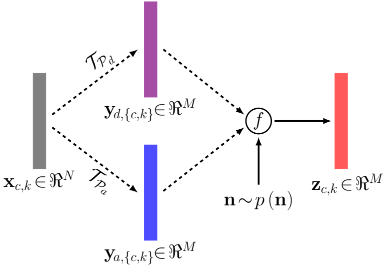
# Proposed Model

**Objective:**

- Learn the discriminative and ambiguous representations with
  - Minimum information loss
  - Supervised discrimination and ambiguization prior
- Obtain the privacy-protected representation by imposing randomness to the learned representations



# Joint Nonlinear Transform Model

$$p\left(\mathbf{Y}_{\{c,k\}}, \mathbf{z}_{c,k} | \mathbf{x}_{c,k}, \mathbf{W}\right) = \int_{\boldsymbol{\theta}} p\left(\mathbf{Y}_{\{c,k\}}, \mathbf{z}_{c,k}, \boldsymbol{\theta} | \mathbf{x}_{c,k}, \mathbf{W}\right) \mathrm{d}\boldsymbol{\theta}$$

- $\mathbf{x}_{c,k} \in \Re^N$: input data
- $\mathbf{z}_{c,k} \in \Re^M$: public (protected) representation
- $\mathbf{Y}_{\{c,k\}} = [\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}] \in \Re^{M \times 2}$: discriminative and ambiguous representations
- $\boldsymbol{\theta} = \{\boldsymbol{\theta}_d, \boldsymbol{\theta}_a\}$: model parameters
- $\mathbf{W} = [\mathbf{W}_d, \mathbf{W}_a], \mathbf{W}_d \in \Re^{M \times N}, \mathbf{W}_a \in \Re^{M \times N}$ : linear transforms

## Model Assumptions:

- $p(\mathbf{x}_{c,k}, \mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta} | \mathbf{W}) = p(\mathbf{x}_{c,k} | \mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta}, \mathbf{W}) p(\mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta} | \mathbf{W})$
- $p(\mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta} | \mathbf{W}) = p(\mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta})$

$$p(\mathbf{z}_{c,k}, \mathbf{Y}_{\{c,k\}}, \boldsymbol{\theta}) = \underbrace{p(\mathbf{z}_{c,k}|\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}, \boldsymbol{\theta})}_{\text{conditional privacy-utility prior}} \underbrace{p(\boldsymbol{\theta}_d, \mathbf{y}_{d,\{c,k\}})}_{\text{discriminative prior}} \underbrace{p(\boldsymbol{\theta}_a, \mathbf{y}_{a,\{c,k\}})}_{\text{ambiguous prior}}$$

$$p(\mathbf{z}_{c,k}|\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}, \boldsymbol{\theta}) \propto \exp(-\frac{1}{\beta_Z}\|\mathbf{z}_{c,k}, -\mathbf{y}_{d,\{c,k\}} - \mathbf{y}_{a,\{c,k\}}\|_2^2) \exp(-\underbrace{L_{p-u}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}})}_{\text{privacy-utility measure}})$$

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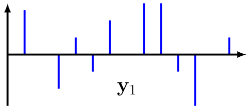
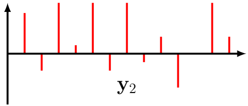
# Support Intersection Based Measures

$$\begin{aligned}
\mathrm{Sim}(\mathbf{y}_1, \mathbf{y}_2) &= \|\mathbf{y}_1^- \odot \mathbf{y}_2^-\|_1 + \|\mathbf{y}_1^+ \odot \mathbf{y}_2^+\|_1 \\
\mathrm{Dis}(\mathbf{y}_1, \mathbf{y}_2) &= \|\mathbf{y}_1^+ \odot \mathbf{y}_2^-\|_1 + \|\mathbf{y}_1^- \odot \mathbf{y}_2^+\|_1 \\
\mathrm{Stg}(\mathbf{y}_1, \mathbf{y}_2) &= \|\mathbf{y}_1 \odot \mathbf{y}_2\|_2^2
\end{aligned}$$



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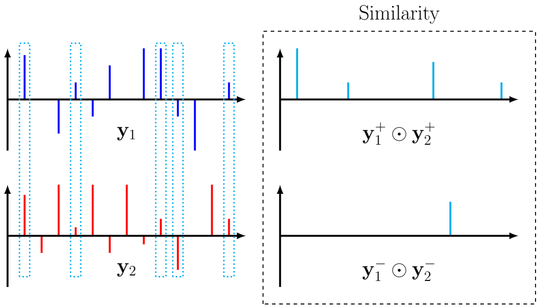

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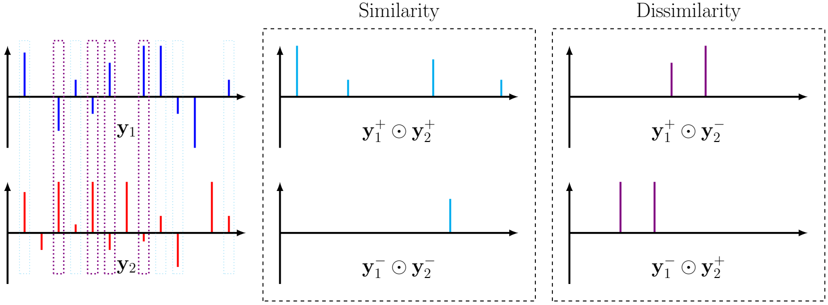


# Functional

$$\boldsymbol{\theta} = \{\boldsymbol{\theta}_d, \boldsymbol{\theta}_a\}, \;\; \boldsymbol{\theta}_d = \{\boldsymbol{\theta}_{d1}, \boldsymbol{\theta}_{d2}\} = \{\overbrace{\{\boldsymbol{\tau}_{d,1}, ..., \boldsymbol{\tau}_{d,D1}\}}^{\text{Dissimilarity Parameters}}, \underbrace{\{\boldsymbol{\nu}_{d,1}, ..., \boldsymbol{\nu}_{d,D2}\}}_{\text{Similarity Parameters}}\}$$

$$\boldsymbol{\theta}_a = \{\boldsymbol{\theta}_{a1}, \boldsymbol{\theta}_{a2}\} = \{\overbrace{\{\boldsymbol{\tau}_{a,1}, ..., \boldsymbol{\tau}_{a,A1}\}}^{\text{Dissimilarity Parameters}}, \underbrace{\{\boldsymbol{\nu}_{a,1}, ..., \boldsymbol{\nu}_{a,A2}\}}_{\text{Similarity Parameters}}\}$$

## Discrimination/Ambiguization Measures:

$$L_p(\boldsymbol{\theta}_p, \mathbf{y}_{p,\{c,k\}}) = \frac{1}{\beta_p} \min_{p1,p2 \in \mathcal{D}} \left( \mathsf{Sim}(\mathbf{y}_{p,\{c,k\}}, \boldsymbol{\tau}_{p,p1}) + \mathsf{Stg}(\mathbf{y}_{p,\{c,k\}}, \boldsymbol{\nu}_{p,p2}) \right)$$

## Discriminative-Ambiguous Measure:

$$L_{p-u}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}) = \frac{1}{\beta_I} \mathsf{Sim}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}) + \frac{1}{\beta_S} \mathsf{Stg}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}})$$

# Supervised Setup

- $V(\mathbf{Y}_d, \mathbf{Y}_a) = \frac{1}{CK} \sum_{c,k} (L_{p-u}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}})$
- $\mathbb{E}[L_d(\boldsymbol{\tau}_{d,c}, \boldsymbol{\nu}_{d,c}, \mathbf{y}_{d,\{c,k\}})] \sim D(\mathbf{Y}_d)$

$$D(\mathbf{Y}_d) = \frac{1}{CK} \frac{1}{\beta_d} \sum_{c} \sum_{c1 \neq c} \sum_{k} \sum_{k1} \left(\mathsf{Sim}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{d,\{c1,k1\}}) + \mathsf{Stg}(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{d,\{c1,k1\}})\right)$$

- $\mathbb{E}[L_a(\boldsymbol{\tau}_{a,c}, \boldsymbol{\nu}_{a,c}, \mathbf{y}_{a,\{c,k\}})] \sim S(\mathbf{Y}_a)$

$$S(\mathbf{Y}_a) = \frac{1}{CK} \frac{1}{\beta_d} \sum_{c} \sum_{k} \sum_{k1 \neq k} \left(\mathsf{Sim}(\mathbf{y}_{a,\{c,k\}}, \mathbf{y}_{a,\{c,k1\}}) + \mathsf{Stg}(\mathbf{y}_{a,\{c,k\}}, \mathbf{y}_{a,\{c,k1\}})\right)$$

# Problem Formulation

$$\min_{\mathbf{Z},\mathbf{Y}_d,\mathbf{Y}_a,\mathbf{W}_d,\mathbf{W}_a} \sum_{p\in\{d,a\}} \Big( \overbrace{\frac{1}{2}\|\mathbf{W}_p\mathbf{X}-\mathbf{Y}_p\|_F^2}^{\text{Nonlinear Transform Error}} + \overbrace{\lambda_{p,1}\sum_{c,k}\|\mathbf{y}_{p,\{c,k\}}\|_1}^{\text{Sparsity Constraint}} \Big)$$

$$+\frac{1}{2}\|\mathbf{Z}-\mathbf{Y}_a-\mathbf{Y}_d\|_F^2 + \overbrace{V(\mathbf{Y}_d,\mathbf{Y}_a)}^{\text{Discriminative}-\text{Ambiguous Constraint}}$$

$$+ \overbrace{D(\mathbf{Y}_d)}^{\text{Discrimination Constraint}} + \overbrace{S(\mathbf{Y}_a)}^{\text{Ambiguization Constraint}}$$

$$+ \overbrace{\Omega(\mathbf{W}_d)+\Omega(\mathbf{W}_a)}^{\text{Linear Map Constraints}}$$

# Learning Algorithm

We propose an iterative, alternating algorithm with five distinct stages:

**(i)** and **(ii):** Estimating discriminative (or ambiguous) representation

**(iii):** Estimating the public (protected) representation

**(iv)** and **(v):** Updating the linear maps

We show that the problems at all stages have an **exact** or **approximate closed-form solutions**.

# Learning Algorithm

## Estimating Discriminative (or Ambiguous) Representation

- Given data samples $\mathbf{X}$, protected representation $\mathbf{Z}$ and current estimate $\mathbf{W}_d$ and $\mathbf{W}_a$
- Discriminative (ambiguous) representation estimation problem is formulated as:

$$\min_{\mathbf{Y}_p} \frac{1}{2}\|\mathbf{W}_p\mathbf{X} - \mathbf{Y}_p\|_F^2 + \frac{1}{2}\|\mathbf{Z} - \mathbf{Y}_d - \mathbf{Y}_a\|_F^2 + V(\mathbf{Y}_d, \mathbf{Y}_a) + D(\mathbf{Y}_d) + S(\mathbf{Y}_a) + \lambda_{p,1}\sum_{c,k}\|\mathbf{y}_{p,\{c,k\}}\|_1, \forall p \in \{d, a\}$$

- Nonlinear Transform Estimation closed-form:

$$\mathbf{y}|_{\{p1,p2\}} = \mathrm{sign}(\mathbf{u}_{p,\{c,k\}}) \odot \max(|\mathbf{u}_{p,\{c,k\}}| - \mathbf{g}_{p,\{c,k\}} - \lambda_{p,1}\mathbf{1}, \mathbf{0}) \oslash (\mathbf{k}_{p,\{c,k\}})$$

# Learning Algorithm

## Estimating the Public (protected) Representation

- Given the estimated discriminative and ambiguous representations, the protected representation is estimated as:

$$\mathbf{z}_{c,k} = f\left(\mathbf{y}_{d,\{c,k\}}, \mathbf{y}_{a,\{c,k\}}, \mathbf{n}\right)$$

- **Objective Perturbation:** Impose random noise $\mathbf{n}$ during the learning phase
- **Output Perturbation:** Impose random noise $\mathbf{n}$ to the final representation

# Learning Algorithm

## Updating the Linear Maps

- Given: data samples $\mathbf{X}$, all representations $\mathbf{Y}_p, p \in \{d, a\}$
- The problem related to the estimation of the linear map $\mathbf{W}_p$, reduces to:

$$\min_{\mathbf{W}_p} \frac{1}{2}\|\mathbf{W}_p\mathbf{X} - \mathbf{Y}_p\|_2^2 + \frac{\lambda_{p,3}}{2}\|\mathbf{W}_p\|_F^2 + \frac{\lambda_{p,4}}{2}\|\mathbf{W}_p\mathbf{W}_p^T - \mathbf{I}\|_F^2 - \lambda_{p,5}\log|\det \mathbf{W}_p^T\mathbf{W}_p|$$

- We use an approximate closed-form solution

# Evaluation

## Computational Efficiency

| AR | | | | | E-YALE-B | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $\mathbf{W}_a$ | | $\mathbf{W}_d$ | | | $\mathbf{W}_a$ | | $\mathbf{W}_d$ | | |
| $\kappa$ | $\mu$ | $\kappa$ | $\mu$ | $t$ | $\kappa$ | $\mu$ | $\kappa$ | $\mu$ | $t$ |
| 155 | 0.002 | 144.98 | 0.002 | 8.094 | 23.32 | 0.001 | 11.96 | 0.001 | 11.24 |

Table: The computational efficiency per iteration $t[sec]$ for the proposed algorithm, the conditioning number $\kappa$ and the expected mutual coherence $\mu$ for the linear maps $\mathbf{W}_a$ and $\mathbf{W}_d$.

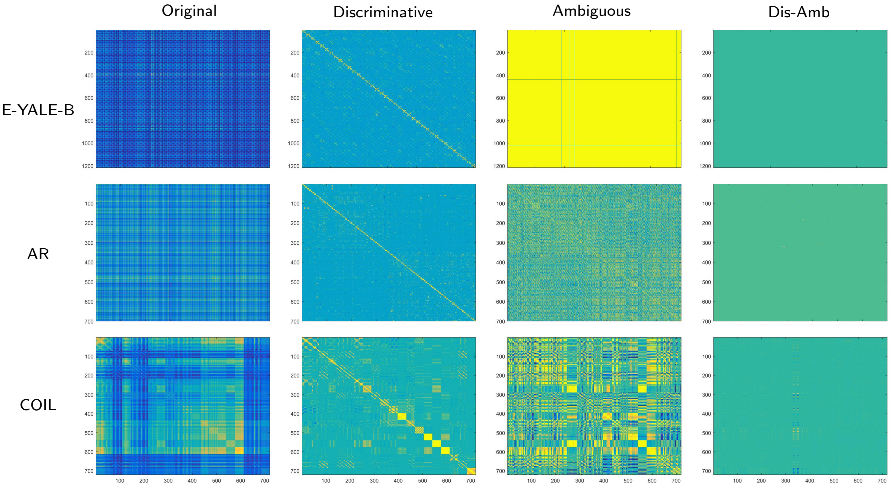
# Evaluation

## k-NN Results

| | COIL | E-YALE-B | AR |
|---|---|---|---|
| Discriminative representation | 99.86 | 94.4 | 88.57 |
| Ambiguous representation | 21.25 | 2.87 | 11.14 |
| Coupled representation | 96.80 | 94.81 | 75.38 |
| Original data | 100 | 81.41 | 84.57 |

Table: The k-NN results on the original data and the assigned NT representations.

# Evaluation




Thank You!